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Real numbers

Part 6 (comparing square roots)

How to compare two numbers including radicals?

❖ **Compare two numbers in form of $\pm\sqrt{a}\ ; a > 0$**

Example: Compare

$\sqrt{12}$ and $\sqrt{15}$

$12 < 15$

So

$\sqrt{12} < \sqrt{15}$

$-\sqrt{21}$ and $-\sqrt{18}$

$21 > 18$

$\sqrt{21} > \sqrt{18}$

So

$-\sqrt{21} < -\sqrt{18}$

$\sqrt{24}$ and $-\sqrt{32}$

$\sqrt{24} > -\sqrt{32}$

Since the positive number is always greater than the negative number.

How to compare two numbers including radicals?

❖ Compare two numbers in form of $\pm a\sqrt{b}\ ; b > 0$

Example 1: if the two numbers include the same radicals.

Compare

$2\sqrt{12}$ and $3\sqrt{12}$

$2 < 3$

So

$2\sqrt{12} < 3\sqrt{12}$

$-5\sqrt{21}$ and $-3\sqrt{21}$

$-5 < -3$

$-5\sqrt{21} < -3\sqrt{21}$

$5\sqrt{26}$ and $-2\sqrt{26}$

$5\sqrt{26} > -2\sqrt{26}$

Since the positive number is always greater than the negative number.

How to compare two numbers including radicals?

❖ **Compare two numbers in form of $\pm a\sqrt{b}$; $b > 0$**

Example : if the two numbers include different radicals.

Compare

$$3\sqrt{15} \text{ and } 2\sqrt{7}$$

$3 > 2$ and $\sqrt{15} > \sqrt{7}$

So,

$$3\sqrt{15} > 2\sqrt{7}$$

How to compare two numbers including radicals?

❖ **Compare two numbers in form of $\pm a\sqrt{b}\ ; b > 0$**

Example 2: if the two numbers include different radicals.

Compare

$3\sqrt{5}$ and $2\sqrt{7}$

Method 1:

Squaring the two numbers:

$\left(3\sqrt{5}\right)^2 = 45$ and $\left(2\sqrt{7}\right)^2 = 28$

$45 > 28$ so, $3\sqrt{5} > 2\sqrt{7}$

Method 2:

$3\sqrt{5} = \sqrt{3^2 \times 5} = \sqrt{45}$

$2\sqrt{7} = \sqrt{2^2 \times 7} = \sqrt{28}$

$45 > 28$ so, $3\sqrt{5} > 2\sqrt{7}$

How to compare two numbers including radicals?

❖ **Compare two numbers in form of $\pm a\sqrt{b}$; $b > 0$**

Example 3: if the two numbers include different radicals.

Compare

$-2\sqrt{11}$ and $-3\sqrt{7}$

Method 1:

Squaring the two numbers $2\sqrt{11}$ and $3\sqrt{7}$:

$(2\sqrt{11})^2 = 44$ and $(3\sqrt{7})^2 = 63$

$44 < 63$ so, $2\sqrt{11} < 3\sqrt{7}$

Then, $-2\sqrt{11} > -3\sqrt{7}$

Method 2:

$-2\sqrt{11} = -\sqrt{2^2 \times 11} = -\sqrt{44}$

$-3\sqrt{7} = -\sqrt{3^2 \times 7} = -\sqrt{63}$

$44 < 63$ so, $\sqrt{44} < \sqrt{63}$

Then $-\sqrt{44} > -\sqrt{63}$

Hence $-2\sqrt{11} > -3\sqrt{7}$

IMPORTANT

- If $a > b$; then $a - b > 0$
- If $a < b$; then $a - b < 0$

❖ **Compare two numbers in form of $a \pm b\sqrt{c}$; $c > 0$**

Example 1:

Compare $2 + \sqrt{3}$ and $2 + \sqrt{5}$

$$\sqrt{3} < \sqrt{5}$$

So

$$2 + \sqrt{3} < 2 + \sqrt{5}$$

$2 + \sqrt{3}$ and $2 - \sqrt{3}$

$$2 + \sqrt{3} > 2$$

$$2 - \sqrt{3} < 2$$

So

$$2 + \sqrt{3} > 2 - \sqrt{3}$$

IMPORTANT

- If $a > b$; then $a - b > 0$
- If $a < b$; then $a - b < 0$

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❖ **Compare two numbers in form of $a \pm b\sqrt{c}$; $c > 0$**

Example 2:

Compare A $= 3 + 2\sqrt{2}$ and B $= 2 + 3\sqrt{2}$

Calculating $A - B$

$A - B = 3 + 2\sqrt{2} - (2 + 3\sqrt{2}) = 3 + 2\sqrt{2} - 2 - 3\sqrt{2} = 1 - \sqrt{2}$

$1 < \sqrt{2}$ so $1 - \sqrt{2} < 0$. Hence, $A < B$

IMPORTANT

- If $a > b$; then $a - b > 0$
- If $a < b$; then $a - b < 0$

❖ **Compare two numbers in form of $a \pm b\sqrt{c}$; $c > 0$**

Example 3:

Compare $A = 2 + 5\sqrt{3}$ and $B = 2 + 4\sqrt{3} + \sqrt{2}$

Calculating $A - B$

$A - B = 2 + 5\sqrt{3} - \left(2 + 4\sqrt{3} + \sqrt{2}\right) = 2 + 5\sqrt{3} - 2 - 4\sqrt{3} - \sqrt{2}$

$= \sqrt{3} - \sqrt{2}$

$\sqrt{3} > \sqrt{2}$ so $\sqrt{3} - \sqrt{2} > 0$. Hence, $A > B$